Dynamic System Simplification Method and Its Application

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Abstract: *The authors present the applications of dynamic system simplification methods in this paper. One simplification method has been discussed with numerical example in which numerator of simplified model is computed using Pade approximations while denominator is determined by using new pole clustering technique. The PI controller has been designed for speed control of DC motor by taking its original transfer function and simplified transfer function. Finally, it is shown that simplified model gives better tuning of the controller. The Matlab software has been used to simulate this problem.*

Keywords: *Dynamic Model, Simplification of model, MATLAB, Controller, Stability.*

1. INTRODUCTION:

The dynamic system simplification plays important roles in engineering and science applications. First advantage of the simplification of the complex models is to provide easy understanding of the system and second advantage is to design efficient controller for the original complex system using its simplified model. Few methods for getting simplified models or reduced order models are already available in the literatures [1-4].
There are numerous applications of simplification methods, which are listed as

- Easy Simulation
- Easy Understanding of the system
- Reduction of computer memory
- Reduction of processing time
- Easy system optimization
- Efficient controller design

The authors suggested a simplification method for large-scale dynamic system based on new pole clustering technique and Pade approximations [3]. The New pole clustering technique is slight modification of pole clustering technique [5] by which dominant pole cluster centres are found. This proposed method is also applicable to unstable system as well. The proposed method has been used for designing a PI controller for speed control of a DC motor. Among the various applications as listed above, one application in controller design has been shown in the paper.

2. DESCRIPTION OF THE SIMPLIFICATION METHOD:

Let the transfer of the high order dynamic system of the order $'n'$ be

$$G(s)=\frac{a_0+a_1s+a_2s^n+...+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+...+b_ns^n} \tag{1}$$

Where a_i and b_i are known scalar constants.

Let the corresponding k^{th} -order simplified model is synthesized as

$$R_k(s)=\frac{N_k(s)}{D_k(s)}=\frac{c_0+c_1s+c_2s^2+...+c_{k-1}s^{k-1}}{d_0+d_1s+d_2s^2+...+d_ks^k} \tag{2}$$

The objective of the problem is to find simplified model as show in equation (2) from (1) such that its behaviour should be as close as possible to original system.
Following two steps are followed to realize a simplified model.

Step-1: Following points should be followed while making the clusters of poles to get reduced model.

- Separate clusters should be made for real and complex poles.
- Poles of imaginary axis have to be retained in the simplified model.

Let, 'r' real poles in one cluster be $\left(p_1, p_2, \ldots, p_r\right)$, then efficient pole cluster centre can be obtained as

$$p_c = p_1 - \left\{\left(\sum_{j=2}^{r} \frac{1}{p_j^{\,2}}\right) \div r^2\right\} \tag{3}$$

Where p_c is the cluster centre.

Similarly, if a cluster contains 'm' number of complex poles such as $[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_{2)}, \ldots..(\alpha_m \pm j\beta_m)]$, then IDM criterion generates a cluster in the form of $A_c \pm jB_c$.

Where $A_c = -\left|\alpha_1\right| - \left\{\left(\sum_{j=2}^{m} \frac{1}{\alpha_j^{\,2}}\right) \div m^2\right\}$ and

$$B_c = -\left|\beta_1\right| - \left\{\left(\sum_{j=2}^{m} \frac{1}{\beta_j^{\,2}}\right) \div m^2\right\}$$

For getting reduced denominator $D_k(s)$ of the simplified model, one of the following cases [6] may be used as

Case-1 if all cluster centres are real, then

$$D_k(s) = (s - p_{c1})(s - p_{c2})\ldots(s - p_{ck}) \tag{4}$$

Case-2 if $(k-2)$cluster centres are real and one pair of cluster centre is complex conjugate then

$$D_k(s) = (s - p_{c1})(s - p_{c2})\ldots(s - p_{c(k-2)})(s - \overset{0}{p}_{c1})(s - \overset{*}{p}_{c1})) \tag{5}$$

Where $\overset{0}{p}_{c1}$ and $\overset{*}{p}_{c1}$ are complex cluster centres i.e. $\overset{0}{p}_{c1} = A_c + jB_c$ and $\overset{*}{p}_{c1} = A_c - jB_c$

Case-3 if all cluster centres are complex conjugates.

$$D_k(s) = (s - \overset{0}{p}_{c1})(s - \overset{*}{p}_{c1})(s - \overset{0}{p}_{c2})(s - \overset{*}{p}_{c2})\ldots(s - \overset{0}{p}_{ck/2})(s - \overset{*}{p}_{ck/2}) \tag{6}$$

Step-2: Determination of the numerator $N_k(s)$ using Pade approximations [3].

The original system can be expanded about $s = 0$ as follows:

$$G(s) = \frac{a_0 + a_1 s + a_2 s^n + \ldots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} = \sum_{i=0}^{\infty} T_i s^i \tag{7}$$

The coefficients 'T_i' are known as time-moments of the original system. The method consists of k -number of equations for finding the numerator coefficients, is so as Pade approximation method [3]. The time-moments T_i can also be calculated through the following equations

$$\left.\begin{aligned} T_0 &= \frac{a_0}{b_0} \\ T_i &= \frac{1}{b_0}\left[a_i - \sum_{j=1}^{i} b_j T_{i-j}\right], i > 0 \\ a_i &= 0, \quad i > n-1 \end{aligned}\right\} \tag{8}$$

The numerator coefficients 'c_i' can be calculated from following Pade equations as

$$\left.\begin{aligned} c_0 &= d_0 T_0 \\ c_1 &= d_0 T_1 + d_1 T_0 \\ &\ldots \\ &\ldots \\ c_{k-1} &= d_0 T_{k-1} + d_1 T_{k-2} + \ldots + d_{k-1} T_0 \end{aligned}\right\} \tag{9}$$

Hence, the reduced numerator can be obtained as

$$N_k(s) = c_0 + c_1 s + c_2 s^2 + \ldots + c_{k-1} s^{k-1} \tag{10}$$

Example-1: Consider an eight-order linear time-invariant system [3].

$$G_8(s)=\frac{N(s)}{D(s)}$$

$$N(s)=18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320$$

$$D(s)=s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320$$

The poles of the system are (-1,-2,-3,-4,-5,-6,-7,-8)
The time-moments are $T_0=1,\quad T_1=1.89,\quad T_2=-3.459$
For getting 2nd –order simplified model, two possible cluster are made as
Cluster-1 : (-1,-2,-3,-4) ; Cluster-2 : (-5,-6,-7,-8)
The p_{c1} and p_{c2} are the cluster centres of the above pole clusters respectively

$$p_{c1}=-\left|-1\right|-\left\{\left(\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\right)\div 4^2\right\}=-1.0264$$

$$p_{c2}=-\left|-5\right|-\left\{\left(\frac{1}{6^2}+\frac{1}{7^2}+\frac{1}{8^2}\right)\div 4^2\right\}=-5.0039$$

Hence, using case-1 of Step-2, $D_2(s)=s^2+6.0303s+5.1360$

Using Step-2 the numerator is obtained as $N_2(s)=15.7373s+5.1360$

Finally, 2nd order simplified model may be written as

$$R_2(s)=\frac{15.7373s+5.1360}{s^2+6.0303s+5.1360}$$ with ISE=0.008286

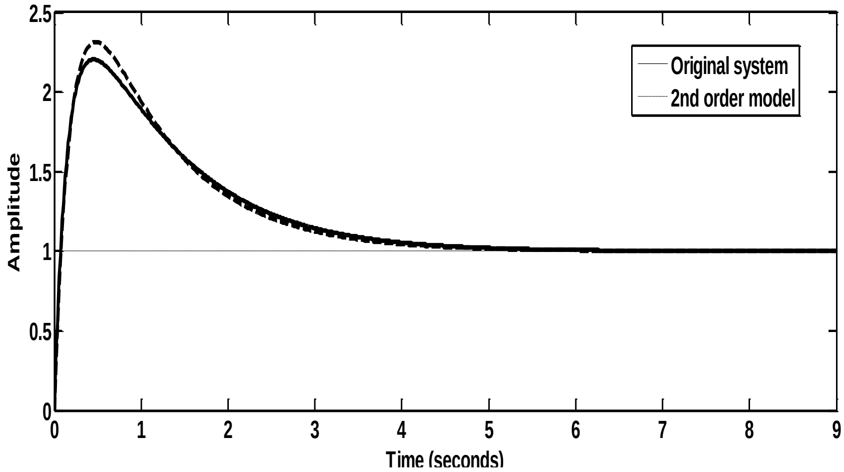

Fig. 1 Step responses comparison

Figure 1 shows better time response matching with the original high order system response. The transient and steady-state region also guarantees appreciable match between the original and simplified step responses. The simplified model can also be compared with the original system using performance index such as Integral Square Error (ISE) [6] and shown in Table 1. Low index means simplified model is better.

Table 1: ISE comparison with the well-known methods

Method of Simplification	Simplified 2nd order models	ISE
Proposed Method	$R_2(s)=\frac{15.7373s+5.1360}{s^2+6.0303s+5.1360}$	0.008286
Mittal et al. [7]	$R_2(s)=\frac{1.9906+7.0908s}{s^2+3s+2}$	0.2689
Pal [8]	$R_2(s)=\frac{40320+151776.576s}{40320+75600s+65520s^2}$	1.6509
Lucas [9]	$R_2(s)=\frac{2+6.7786s}{2+3s+s^2}$	0.2792

Example-2: The transfer function of DC motor is defined as given below and first order simplified model is required.

$$\frac{W(s)}{V(s)} = \frac{0.01}{0.005s^2 + 0.06s + 0.1001}$$

Where $W(s)$: Speed of DC motor (rad/sec)

$V(s)$: Armature Voltage (Volts)

Now using the proposed method, simplified model is obtained as

$$\frac{W(s)}{V(s)} = \frac{0.200299}{s + 2.0050}$$

The step response comparison of the original and simplified models is shown in figure.

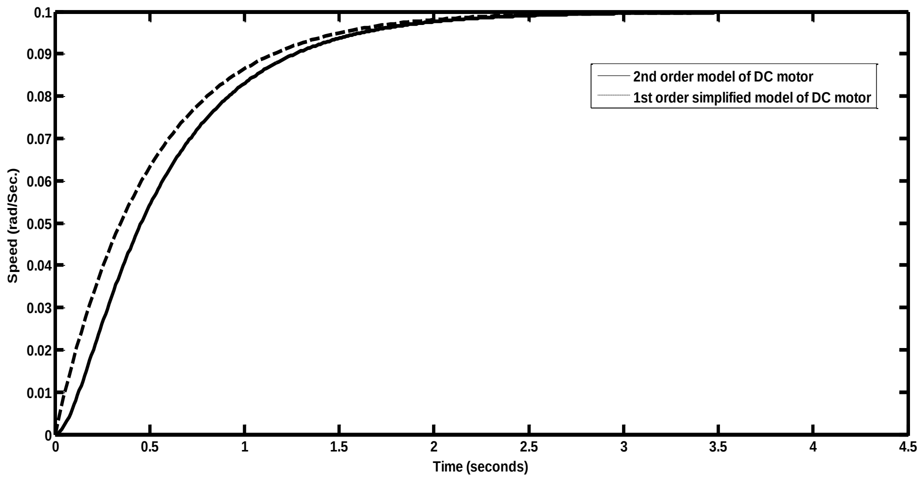

Fig.2 Step response comparison of DC motor

2. APPLICATION OF SIMPLIFIED MODEL FOR CONTROLLER DESIGN:

In this section, the simplified and original model of the DC motor are considered to design Proportional and Integral (PI) controller to control the speed of the DC motor using MATLAB software.

The following objectives are required as

1- Final Value of the response should be 1, as here we can see as 0.1

2- Settling Time should be less than 2 sec.

3- Response should not have peak /oscillations.

Step-1: PI Controller Design using MATLAB simulink

The PI controller is tuned to get desired response with the help of MATLAB Simulink model. The Simulink model is made to design the PI controller using MATLAB software.

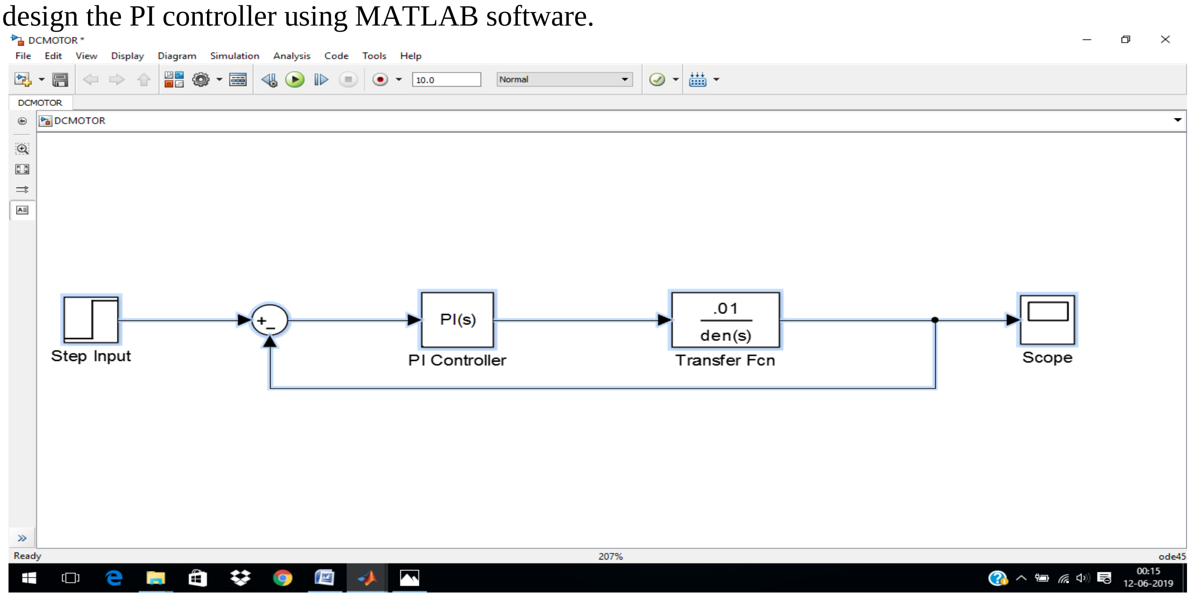

Fig.3 Simulink Model with PI Controller and original DC motor model

The speed response of 2nd order model of DC motor with PI Controller is shown

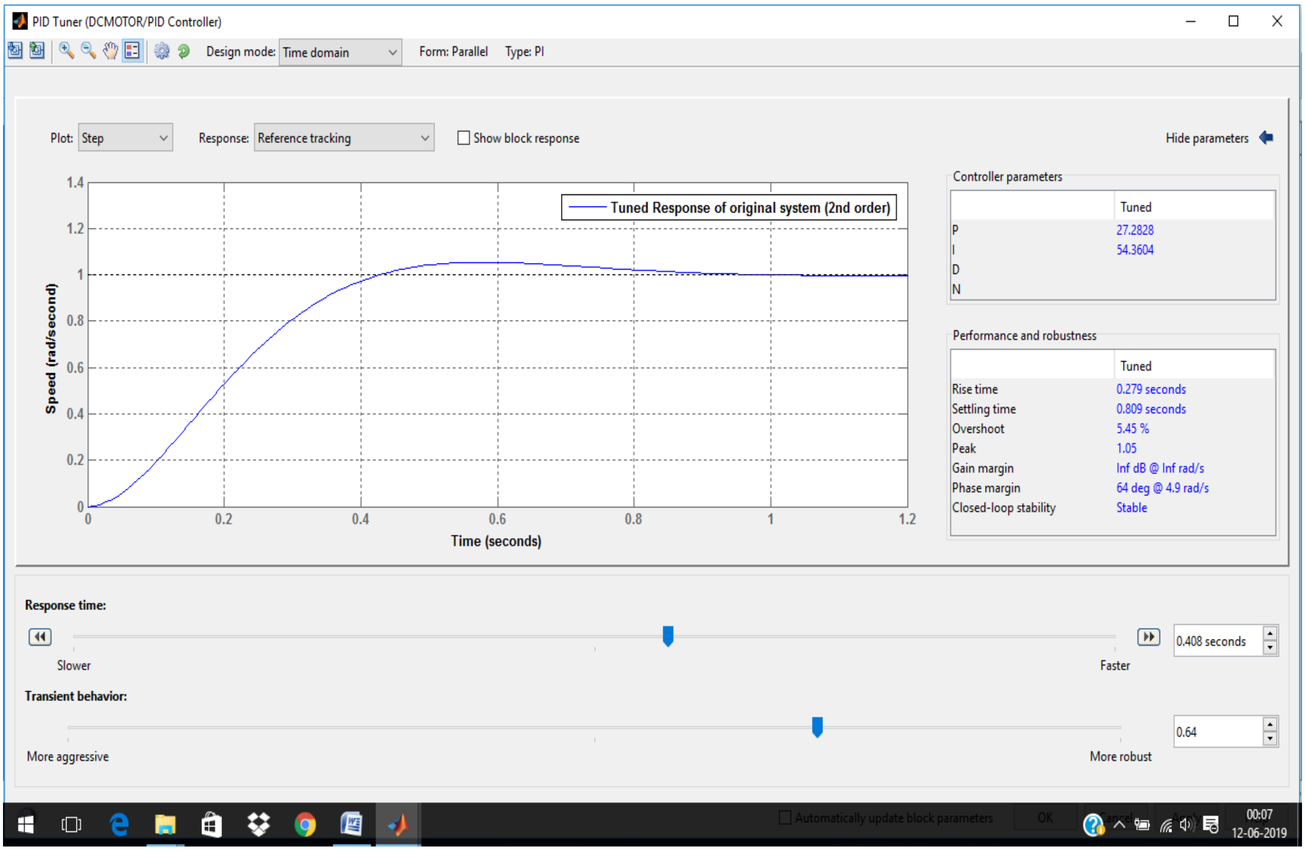

Fig. 4 Closed loop response of DC motor with PI Controller

Step-2: Design of PI controller for simplified DC motor model

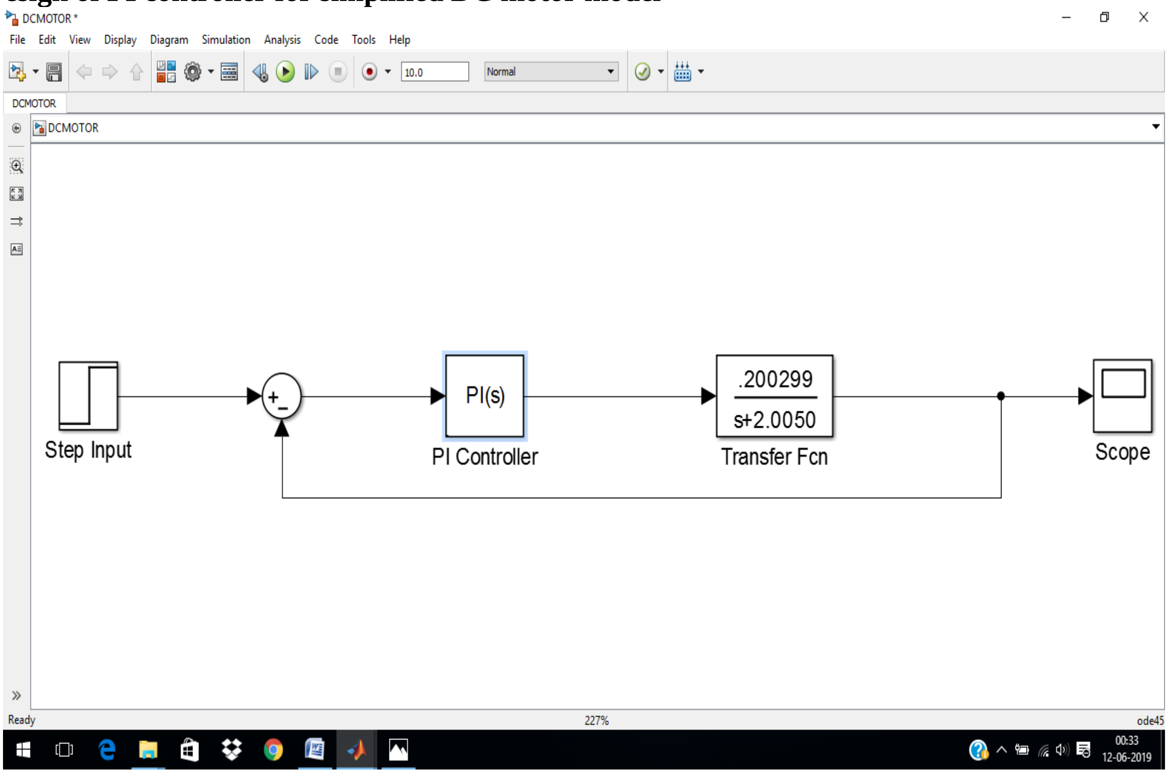

Fig. 5 Simulink model for simplified DC motor with PI controller

The controlled speed response of DC motor (1st order simplified model) with PI controller is shown in screenshot

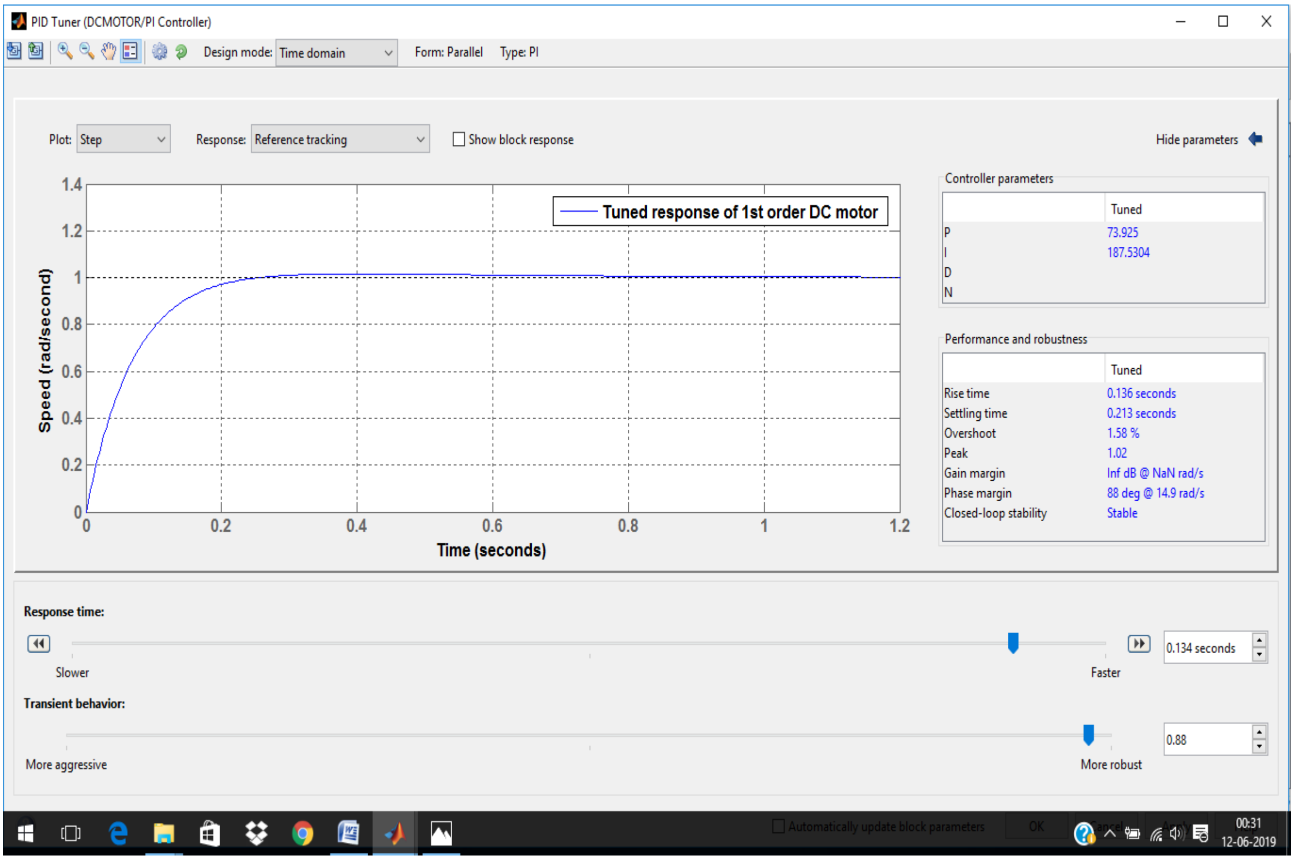

Fig. 6 PI controller response for simplified DC motor model

PI controller responses can be plotted in the same figure for comparison purposes.

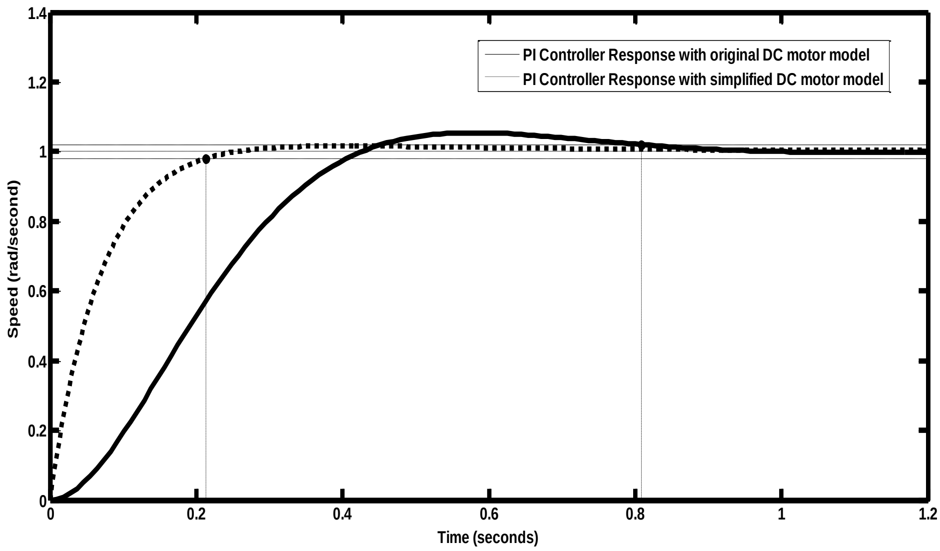

Fig. 7 Comparison of PI controller performances with original and simplified DC motor

The performance specifications are compared and tabulated in Table 1 from which the results may be concluded easily.

Table 2: Performance comparison of PI controller with original and simplified model

S.N	PI Controller	Rise Time	Settling Time	Peak overshoot
1	With original DC motor model	0.279	0.803	1.05
2	With simplified DC motor model	0.136	0.213	1.02

From table 2, it can be observed that simplified model based controller gives more acceptable specifications/results.

3. RESULTS AND DISCUSSION:

In this paper, two original dynamic systems are considered for validation of the proposed simplification method. The simplified models are compared graphically and also compared through performance index. The large-scale system is simplified to 2nd order model in example 1 while 2nd order model is simplified to 1st order model in example 2. From the both examples, it is clear that the proposed simplification method is efficient and retains the important properties of the original system. Finally, the application of the simplified model for PI controller design is shown and results are compared, if PI controller is designed for original model of DC motor. From Table 2, it is very much clear that PI controller performance based on simplified model is better than performance based on original 2nd order DC motor model.

4. CONCLUSION:

The authors suggested a new pole clustering technique for simplification of a linear dynamic system. The viability of the proposed method has been tested on numerical examples taken from literature. The application of the simplification method has been shown by designing a PI controller on the simplified model of DC motor for controlling its speed. Finally it may be concluded that simplified model of complex dynamic system plays important role to design an efficient control system.

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